

## Short Note

### “Unexpected” shear-wave behavior

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#### INTRODUCTION

In a previous study, we inverted shear-wave birefringence observations from an azimuthal vertical seismic profile (VSP) experiment conducted at the Conoco Borehole Test Facility, Oklahoma (Horne and MacBeth, 1994; Horne, 1995). Our results indicate that the observations can be interpreted in terms of two distinctly different transversely isotropic (TI) models (Figure 1). The first model predicts the symmetry axis to be at N165°E and dipping 10° to the northwest. This orientation coincides with geological information relating to the fracture system that strikes between N50°E and N75°E (Queen and Rizer, 1990). Thus, this first model is consistent with a priori information, so that a possible source of the anisotropy can be identified. However, the second model derived from the inversion results suggests the symmetry axis to be at N200°E and dipping 30° to the southwest. If we interpret this result in terms of an equivalent medium resulting from aligned cracks or fractures, then this inferred crack-fracture strike would lie in a direction conflicting with the a priori measurements. The bimodal nature of this solution can be readily understood if we examine the shear-wave behavior for the different models shown in Figure 2. In this plot, the symmetry axis is chosen to be the  $x_1$ -direction. If we consider the near-vertical propagation directions that are typically measured in VSP experiments, it can be seen that the  $qS1$  polarizations lie either perpendicular (model 1) or parallel (model 2) to the symmetry axis. Since these polarizations are usually interpreted in terms of aligned crack-fracture systems, the inferred strike would lie in the  $x_2x_3$  plane for model 1 and the  $x_1x_3$  plane for model 2. This interpretation is completely incorrect for model 2, since this inferred alignment is actually orthogonal to the alignment implied by the symmetry of the TI system. This situation represents a worrying aspect to the interpretation of shear-wave surveys used to characterize crack-fracture systems. The question that we address is whether anisotropic materials that possess properties

similar to those of model 2 can be constructed from equivalent media resulting from cracks or fractures. We also consider other sources of anisotropy that may lead to this behavior.

#### TRANSVERSE ISOTROPY AND EQUIVALENT MEDIA

For TI materials, the quasi-shear waves are polarized in a plane that is either parallel or perpendicular to the symmetry axis. We adopt the notation of Crampin (1981) and refer to the shear wave polarized in the plane perpendicular to the symmetry axis as  $qSR$  and the shear wave polarized parallel to the symmetry axis as  $qSP$ . If the symmetry axis lies in the  $x_1$ -direction, then the elastic stiffness matrix is

$$\begin{pmatrix} c_{11} & c_{12} & c_{12} & & & \\ & c_{22} & c_{22} - 2c_{44} & & & \\ & & c_{22} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{55} \end{pmatrix}. \quad (1)$$

The vertical velocities for the  $qSP$  and  $qSR$  shear modes are

$$V_{qSP} = \sqrt{\frac{c_{55}}{\rho}}, \quad (2)$$

$$V_{qSR} = \sqrt{\frac{c_{44}}{\rho}}. \quad (3)$$

To obtain the type of behavior exhibited in model 2, we require the first shear-wave arrival propagating in the vertical direction to be the  $qSP$  mode. This condition can be shown to impose an inequality on the elastic constants, written as

$$c_{44} < c_{55}. \quad (4)$$

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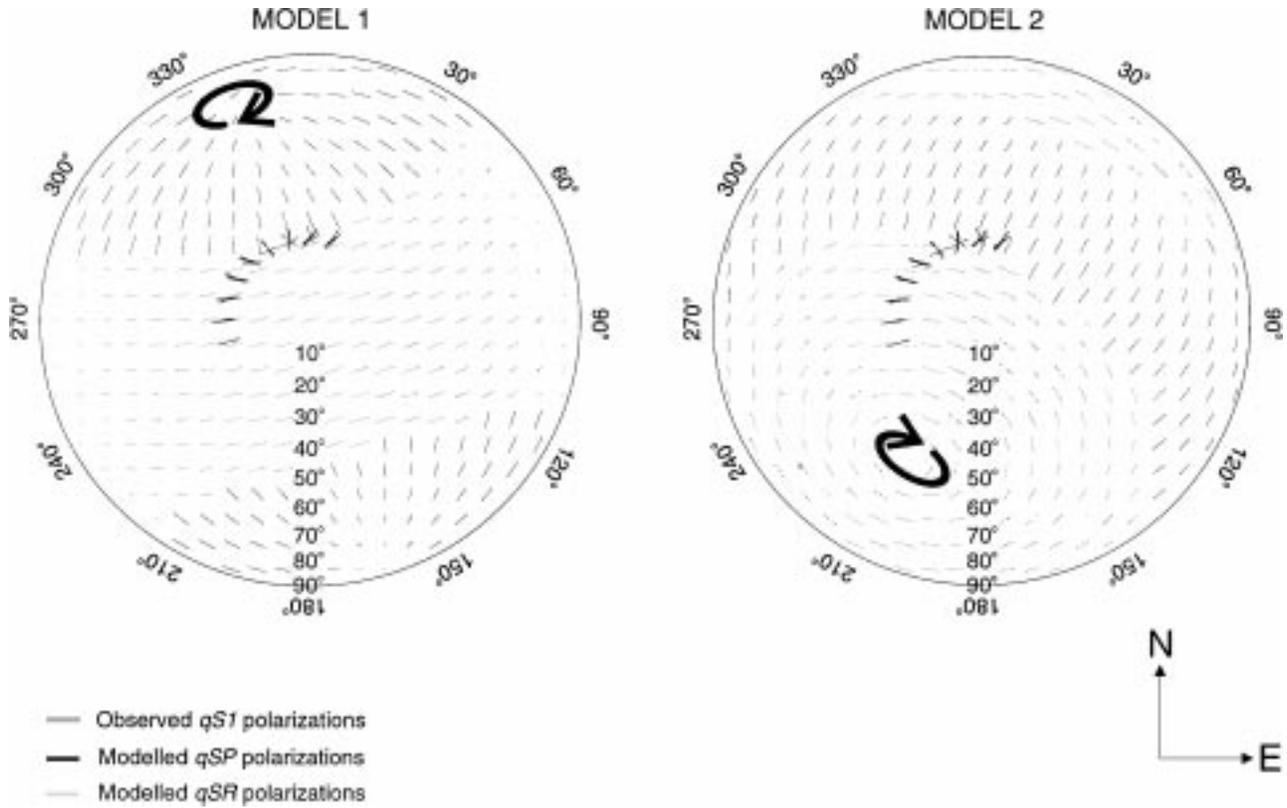


FIG. 1. Equal-area plots showing the fast-shear-wave polarization for two models derived by inversion of shear-wave splitting observations from an azimuthal VSP experiment. This indicates the bimodal nature of the solution. Model 1 (a) is characterized by the fast-shear-wave arrival being polarized in the plane normal to the symmetry axis (indicated by the curved arrow). Model 2 (b) is characterized by the fast-shear-wave arrival being polarized in the plane containing the symmetry axis.

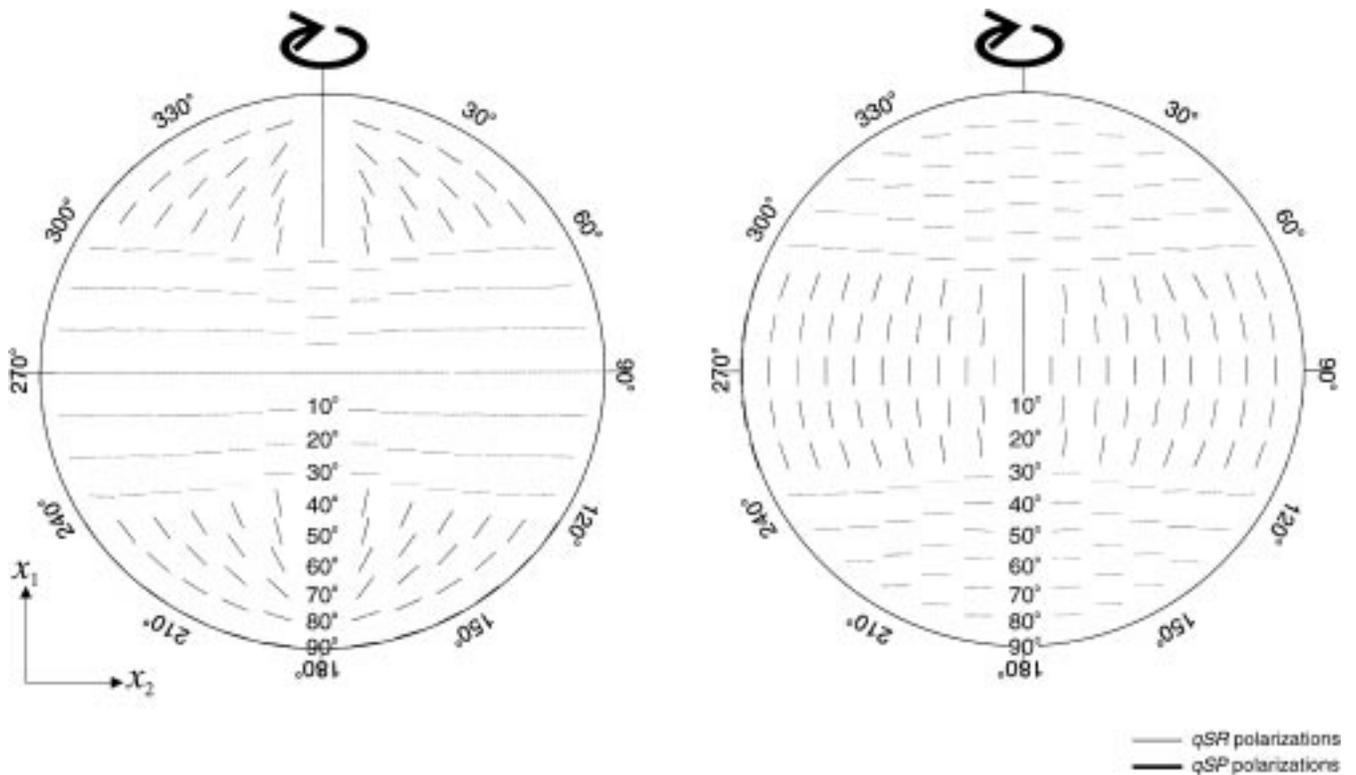


FIG. 2. TI media with the symmetry axis aligned in the  $x_1$ -direction. (a) The fast shear-wave arrival is polarized in the plane containing the symmetry axis ( $qSP$ ). (b) The fast-shear-wave arrival is polarized in the plane normal to the symmetry axis ( $qSR$ ).

Alternatively, this may be expressed conveniently using the Thomsen (1986) parameter  $\gamma$  as

$$\gamma < 0, \tag{5}$$

where  $\gamma$  is defined as  $\gamma = (c_{44} - c_{55})/2c_{55}$  for a TI material with the symmetry axis aligned in the  $x_1$ -direction. We now investigate whether equivalent anisotropic materials satisfying this inequality can be constructed from the following systems (Figure 3):

- 1) aligned oblate spheroids
- 2) joint-fracture anisotropy
- 3) fine-layering anisotropy
- 4) coplanar normal cracks
- 5) aligned prolate spheroids
- 6) intrinsic anisotropy.

**Aligned oblate spheroids**

Oblate spheroids take the geometric form of thin penny-shaped cracks (aspect ratios less than one). Equivalent media constructed from systems of aligned oblate spheroids are described in Hudson (1980). Assuming that the Hudson crack model (1980) accurately reproduces shear-wave behavior, we may relate the inequality relation  $c_{44} < c_{55}$  to the crack

parameters. The second-order formulations of Hudson (1980) can be written as

$$c = c_0 + c_1 + c_2, \tag{6}$$

where  $c$  is the equivalent anisotropic stiffness matrix,  $c_0$  is the isotropic elastic constant of the background matrix, and  $c_1$  and  $c_2$  are the first- and second-order contributions, respectively. The first- and second-order perturbations obtained from the Hudson model (1980) for the elastic constants  $c_{44}$  and  $c_{55}$  are

$$c_{44}^1 = 0, \tag{7}$$

$$c_{44}^2 = 0, \tag{8}$$

$$c_{55}^1 = -\epsilon\mu u_{33}, \tag{9}$$

$$c_{55}^2 = \frac{\epsilon^2}{15}\mu \frac{(3\lambda + 8\mu)}{(\lambda + 2\mu)} u_{33}^2, \tag{10}$$

where  $\epsilon$  is the crack density, defined to be the number of cracks per unit volume and  $\lambda$  and  $\mu$  are the Lamé constants of the isotropic uncracked solid. The term  $u_{33}$  is defined by boundary conditions across the cracks, which for dry and fluid-filled inclusions is given by

$$u_{33} = \frac{16}{3} \frac{(\lambda + 2\mu)}{(3\lambda + 4\mu)}. \tag{11}$$

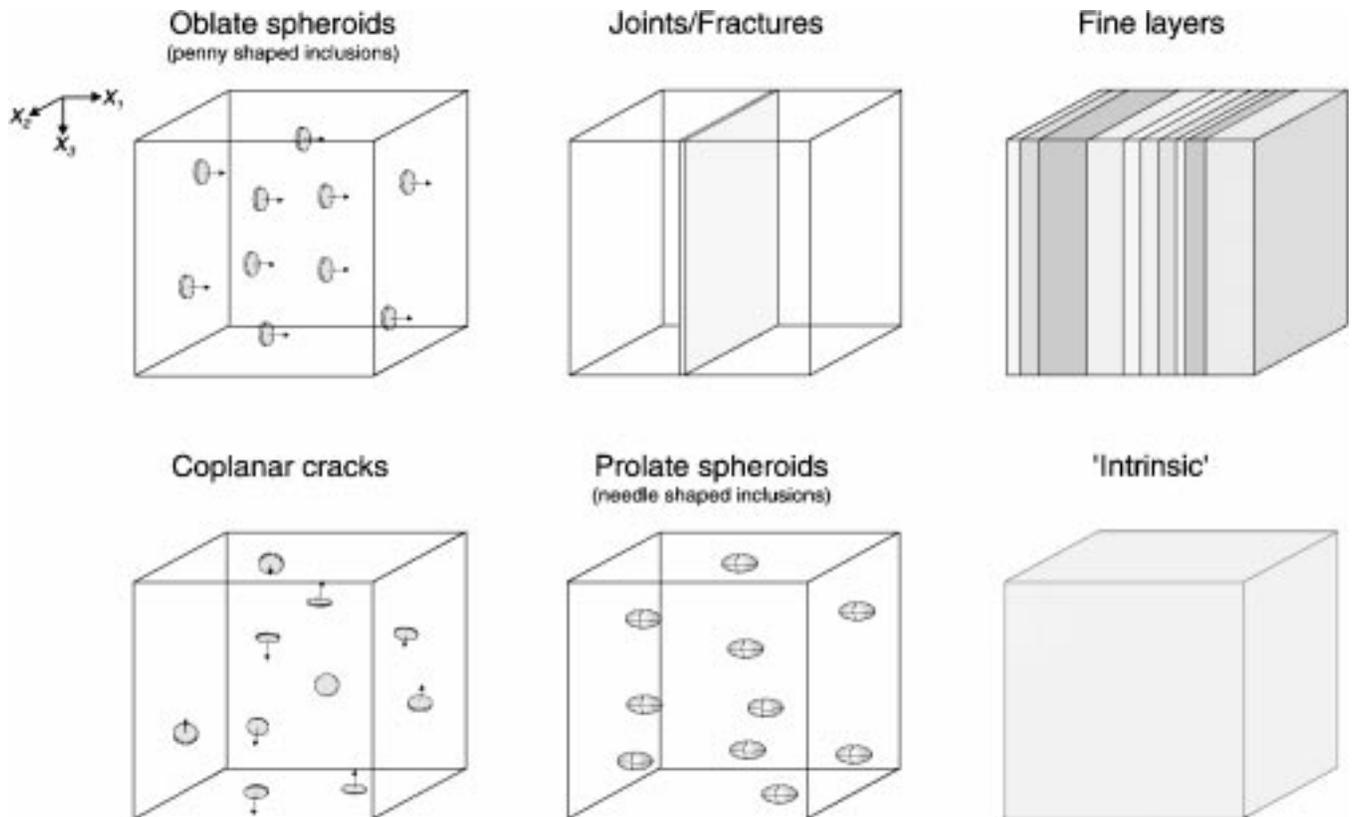


FIG. 3. Schematic illustration of some generating mechanisms of transverse isotropy that are considered in this paper as possible sources of the unexpected shear-wave behavior.

Inserting these quantities into the inequality  $c_{44} < c_{55}$  gives

$$\frac{V_p}{V_s} < \sqrt{\frac{90 + 32\epsilon}{135 - 48\epsilon}}. \quad (12)$$

This inequality cannot be satisfied for any realistic values of  $V_p$ ,  $V_s$ , and  $\epsilon$ , so materials with the “unexpected” shear-wave behavior described earlier are unlikely to exist as a result of aligned crack systems.

### Joints or fractures

A similar result might be expected for equivalent media constructed from aligned joints or fractures, since the formulations are similar (Schoenberg and Sayers, 1995). Confirmation is obtained using the Schoenberg-Muir decomposition (Schoenberg and Douma, 1988). This technique allows the anisotropic elastic tensor to be decomposed into a combination of a joint-fracture compliance and a “background” compliance. This method constructs a diagonal  $3 \times 3$  joint-fracture compliance matrix  $\mathbf{Z}$ . The diagonal elements of this matrix are the compliances normal  $Z_n$  and tangential  $Z_t$  to the fracture plane. For joints or fractures aligned normal to the  $x_1$ -direction, the normal fracture compliance is given by Hood (1991) as

$$Z_t = \frac{1}{c_{55}} - \frac{1}{c_{44}}. \quad (13)$$

The  $Z_t$  term will be negative in the case that  $c_{44} < c_{55}$ . This violates the positive nature of the strain-energy function, which requires all diagonal elements of the compliance matrix to be positive (Helbig, 1994). Thus, materials with this unusual shear-wave behavior are unlikely to be due to joint-fracture-induced anisotropy.

### Fine-layering anisotropy

We now consider whether equivalent media resulting from fine layering can satisfy the inequality  $c_{44} < c_{55}$ . We use the fine-layering formulations of Postma (1955) for which a TI medium can be constructed from the isotropic Lamé constants and thicknesses of two alternating layers. The relevant formulations for a biperiodic system, assuming the symmetry axis to be aligned along the  $x_1$ -axis (for consistency), are

$$c_{44} = \frac{\mu_1 d_1 + \mu_2 d_2}{d_1 + d_2}, \quad (14)$$

$$c_{55} = \frac{(d_1 + d_2)\mu_1\mu_2}{d_1\mu_2 + d_2\mu_1}, \quad (15)$$

where the subscripts identify the layers and  $d$  is the individual layer thickness. Substituting these into the inequality leads to

$$(\mu_1 - \mu_2)^2 < 0. \quad (16)$$

Thus, this simple system of a biperiodic layering medium cannot account for the type of behavior exhibited in model 2. However, the generality of this result when extended to other finely layered systems is not considered in this short note.

### Coplanar normal cracks

A crack system is coplanar if the normal cracks are constrained to lie in a common plane but are otherwise randomly oriented. The Hudson crack model (1980) can be adapted to calculate elastic stiffness for such a situation. The first-order perturbations are

$$c_{44}^1 = -\frac{\epsilon\mu}{2}(u_{33} + u_{11}), \quad (17)$$

$$c_{55}^1 = -\frac{\epsilon\mu}{2}(u_{33}). \quad (18)$$

For an anisotropic medium composed of a system of coplanar cracks to satisfy the inequality  $c_{44} < c_{55}$ ,  $u_{11}$  must be positive. Since  $u_{11}$  is always positive, the inequality is always satisfied. Therefore, materials of this type will exhibit this type of behavior.

### Aligned prolate spheroids

Prolate spheroids are needle-shaped objects with aspect ratios greater than one. Equivalent-medium formulations for these systems have been constructed by Hudson (1994) and Nishizawa (1982). Simple expressions for the stiffnesses  $c_{44}$  and  $c_{55}$  cannot be found using either of these schemes. However, materials satisfying the inequality  $c_{44} < c_{55}$  can readily be derived using realistic parameters. A specific example is given in Tables 1 and 2.

### Intrinsic anisotropy

We consider intrinsic anisotropy as an interpretation of the results. Thomsen (1986) tabulates an extensive list of intrinsically anisotropic materials. Three of the sedimentary materials in this list satisfy the inequality  $c_{44} < c_{55}$ , namely, two sandstone samples and a mudshale (Table 3). Some sandstones can be modeled as a system of aligned ellipsoidal particles cemented in an isotropic matrix. Such a system is essentially the same

**Table 1. Modeling parameters for the intrinsic-anisotropy sample composed of ellipsoidal particles.**

| Parameter                              | Value |
|----------------------------------------|-------|
| Inclusion                              |       |
| $\lambda$ (MPa)                        | 24.5  |
| $\mu$ (MPa)                            | 27.9  |
| Matrix                                 |       |
| $\lambda$ (MPa)                        | 20.8  |
| $\mu$ (MPa)                            | 24.1  |
| Density $\rho$ (g/cm <sup>3</sup> )    | 2.7   |
| Inclusion density (g/cm <sup>3</sup> ) | 0.5   |
| Aspect ratio                           | 1.5   |

**Table 2. Elastic stiffness for the intrinsic-anisotropic sample defined in Table 1.**

| Stiffness | Value (MPa) |
|-----------|-------------|
| $c_{11}$  | 102.2       |
| $c_{22}$  | 101.3       |
| $c_{44}$  | 35.0        |
| $c_{55}$  | 35.2        |
| $c_{12}$  | 31.3        |

**Table 3. Materials given by Thomsen (1986) for which  $c_{44} < c_{55}$ .**

| Material             | $V_P$<br>(km/s) | $V_S$<br>(km/s) | $\rho$<br>(g/cm <sup>3</sup> ) | $\epsilon$ | $\delta$ | $\gamma$ |
|----------------------|-----------------|-----------------|--------------------------------|------------|----------|----------|
| Calcareous sandstone | 5.5             | 3.2             | 2.7                            | 0.000      | -0.264   | -0.007   |
| Mudshale             | 5.1             | 3.0             | 2.7                            | 0.010      | 0.012    | -0.005   |
| Sandstone            | 4.9             | 2.9             | 2.7                            | 0.033      | 0.040    | -0.019   |

as the system of aligned prolate spheroids described earlier. Therefore, the unusual shear-wave behavior for these intrinsically anisotropic sandstones may be the result of the preferential orientation of the sand particles.

#### DISCUSSION AND CONCLUSIONS

In this short note, we have discussed whether TI equivalent media can be constructed from realistic anisotropic mechanisms for which the  $qSP$  arrival is the fast shear wave. We have shown that crack, joint-fracture, and fine-layering systems cannot model this behavior. We also have shown that aligned prolate spheroids, coplanar cracks, and some intrinsically anisotropic systems can exhibit this unexpected shear-wave behavior. This result is comforting, since the interpretation of the fast shear wave in terms of the crack-fracture strike still is generally valid if the cause of the anisotropy is cracks or fractures.

If the anisotropy is caused by partly aligned prolate spheroid particles, as is likely to be the case for sandstones, then the direction of the fast shear wave still can be interpreted as the direction of maximum permeability, as in the case of crack-fracture-induced anisotropy. Alluvial sandstones consisting of ellipsoidal particles typically are deposited so that the long axis lies parallel to the fluid-flow direction, with the dip direction pointing upstream, as illustrated in Figure 4 (Lené and Owen, 1969; Prince, Ehrlich, and Anguy, 1995). This implies that the symmetry axis of the equivalent anisotropic medium will be aligned in a subhorizontal plane, so the behavior exhibited by model 2 can be expected to occur in sandstone formations. To resolve these situations, a priori information must be taken into account and care must be taken to avoid oversimplifications when interpreting shear-wave polarizations.

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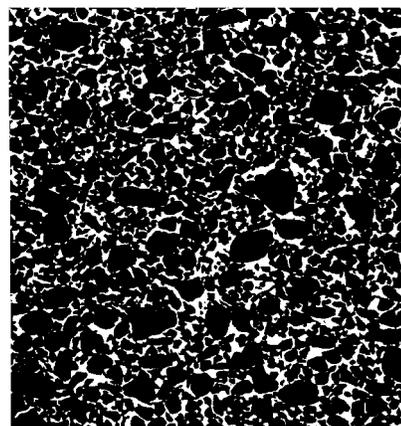


FIG. 4. Photomicrograph of a poorly sorted sandstone showing an alignment of roughly ellipsoidal particles. (After Prince et al., 1995)

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