Linear-Transform Techniques for Analyzing Shear-Wave Splitting in Four-Component Seismic Data

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Summary

Most techniques for analyzing shear-wave splitting employ rotation procedures and tend to be computing intensive. Here, we present a fast linear-transform technique for analyzing shear-wave splitting in four-component seismic data (two source orientations recorded by two receivers). We transform the data by four linear transforms so that the shear-wave motion is linearized in a wide variety of conditions. Many properties of shear-waves can be easily estimated from the transformed components.

The technique allows various attributes to be measured, including the polarizations of faster split shear-wave, and the time delays between faster and slower split shear-waves, as well as allowing the time series of the faster and slower split shear-waves to be separated deterministically. In addition, with minimum assumptions, the geophone orientations can be estimated for zero-offset VSPs, and the polarizations of the slower split shear-waves can be estimated for offset VSPs. The time series of the split shear-waves can be separated before stack for reflection surveys. The technique has been successfully applied to a number of field VSPs and reflection data sets. Applications to a zero-offset VSP, an offset VSP, and a reflection dataset are presented to illustrate the technique.

Introduction

In a homogeneous crack-induced anisotropic medium, caused, say, by a single set of parallel vertical cracks in an isotropic matrix rock, the recorded four-components can be expressed in terms of the properties of the uncracked matrix and the properties of the faster and slower split shear-waves. Thus, in principle, the properties of the matrix and the faster and slower split shear-waves can be determined from the recorded components.

Among the earliest studies of techniques for solving this inversion problem was the use of rotation analysis by Alford (1986) and Thomsen (1988). A number of similar rotation procedures have been developed by Nicoletis et al. (1988), Murtha (1989), Macbeth and Crampin (1991), and others. Most of these techniques employ rotation scanning procedures and tend to be computing intensive.

In an attempt to overcome some of these problems, we first developed complex component analysis (Li and Crampin, 1990) and now we propose a new fast technique, which we call the linear-transform technique, for analyzing shear-wave splitting in four-component seismic data.

Basic definitions

Figure 1a shows the source geometry, and Figure 1b shows the surface projection of the geophone geometry, where **e1** and **e2** are two unit vectors representing the

directions of faster and slower split shear-waves. The medium between the source and the **first geophone**, and between successive geophones, is assumed to be homogeneous with a uniform distribution of cracks.

We define the principal time series $qS1(t)$ and $qS2(t)$ of the faster and slower split shear-waves, respectively, as the time series received at a geophone position when a source vector **F** is polarized along **e1** and **e2**, respectively, with signature **F(i)**. The concept of the principal time series was introduced by Alford (1986) and Thomsen (1988). Here, we give an alternative geometrical definition. We introduce two transformed time series $V1(t)$ and $V2(t)$ as the sum and difference, respectively, of the principal time series $qS1(t)$ and $qS2(t)$:

$$\begin{aligned} V1(t) &= qS1(t) + qS2(t); \\ V2(t) &= qS1(t) - qS2(t). \end{aligned} \quad (1)$$

We define the four-component time series as $s_{ij}(t)$ recorded from x - and y -sources ($i = 1, 2$) on x - and y -geophones ($j = 1, 2$). Following a similar approach to Alford (1986) and Thomsen (1988), The four components, $s_{ij}(t)$, can be written as:

$$\begin{aligned} s_{11}(t) &= [qS1(t) \sin\beta' \cos\alpha - qS2(t) \sin\alpha' \cos\beta] / \sin(\beta' - \alpha'); \\ s_{21}(t) &= [qS1(t) \sin\beta' \sin\alpha - qS2(t) \sin\alpha' \sin\beta] / \sin(\beta' - \alpha'); \\ s_{12}(t) &= [-qS1(t) \cos\beta' \cos\alpha + qS2(t) \cos\alpha' \cos\beta] / \sin(\beta' - \alpha'); \\ s_{22}(t) &= [-qS1(t) \cos\beta' \sin\alpha + qS2(t) \cos\alpha' \sin\beta] / \sin(\beta' - \alpha'), \end{aligned} \quad (2)$$

where α , β , α' and β' are the angles in Figure 1a and 1b. In cases where geophones and sources are orientated in the same direction ($\alpha = \alpha'$; $\beta = \beta'$) and the two shear-waves are orthogonal to each other ($\beta' - \alpha' = \pi/2$), these equations will reduced to Thomsen's equations (7), (8) and (9) (Thomsen, 1988).

Linear transform techniques

The four linear transforms are:

$$\xi(t) = s_{11}(t) - s_{22}(t); \quad (3)$$

$$\eta(t) = s_{21}(t) + s_{12}(t); \quad (4)$$

$$\zeta(t) = s_{11}(t) + s_{22}(t); \text{ and} \quad (5)$$

$$\chi(t) = s_{12}(t) - s_{21}(t). \quad (6)$$

[Linear transforms (3) and (4) were first used by Li and Crampin (1990) to calculate orientation logs in reflection surveys.] The results of the transformed components for orthogonal and non-orthogonal split shear-waves are discussed separately.

ORTHOGONAL SPLIT SHEAR-WAVES: The two split shear-waves in distributions of parallel vertical cracks can be assumed to be orthogonal to each other at vertical incidence,

as in a zero-offset VSP. When the orientation of the **downwell** geophones is unknown, we can write $\beta = \pi/2 + a$ and $\beta' = \pi/2 + a'$, and the transformed components can be written as:

$$\begin{aligned}\xi(t) &= [qS1(t) - qS2(t)] \cos(\alpha + \alpha'); \\ \eta(t) &= [qS1(t) - qS2(t)] \sin(\alpha + \alpha');\end{aligned}\quad (7)$$

$$\begin{aligned}\zeta(t) &= [qS1(t) + qS2(t)] \cos(\alpha - \alpha'); \text{ and} \\ \chi(t) &= -[qS1(t) + qS2(t)] \sin(\alpha - \alpha').\end{aligned}\quad (8)$$

Equations (7) and (8) show time series $V1(t) = qS1 + qS2$ and $V2(t) = qS1 - qS2$ are separated in the transformed components as linear motion, so we can easily estimate $V1$ and $V2$, and $a + a'$ and $a - a'$ from the transformed components. Hence, $qS1 = (V1 + V2)/2$ and $qS2 = (V1 - V2)/2$, and a and a' can be determined, and the orientation of the geophone can be estimated from $a - a'$.

NON-ORTHOGONAL SPLIT SHEAR-WAVES: In off-vertical incidence, the polarizations of the faster and slower split shear-wave in the horizontal plane may not be orthogonal. Such cases include **offset-VSPs**, and pre-stack data in reflection surveys. In all cases of non-orthogonality, it is reasonable to assume that the orientation of geophone is same as the source, that is, $a = a'$, and $\beta = \beta'$, the results of the transformed components can be written as:

$$\begin{aligned}\xi(t) &= [qS1(t) - qS2(t)] \sin(\alpha + \beta)/\sin(\beta - \alpha); \\ \eta(t) &= -[qS1(t) - qS2(t)] \cos(\alpha + \beta)/\sin(\beta - \alpha);\end{aligned}\quad (9)$$

$$\begin{aligned}\zeta(t) &= qS1(t) + qS2(t); \text{ and} \\ \chi(t) &= -[qS1(t) - qS2(t)] \cos(\beta - \alpha)/\sin(\beta - \alpha).\end{aligned}\quad (10)$$

Again time series $V1 = qS1 + qS2$ and $V2 = qS1 - qS2$ are separated: $V1$ is directly given by component $\zeta(t)$, and $V2$ is a linear motion as in equation (9). Thus, the faster split shear-wave $qS1$ and polarization angle a , and the slower split shear-wave, $qS2$ and angle β can also be determined for non-orthogonal split shear-waves. Angle $\beta - \alpha$ is the angle between the faster and slower split shear-wave and is a measure of the orthogonality of the split shear-waves.

Verification of the technique

To verify this technique, we demonstrate the linearity of the transformed time series in field **VSPs**. Figure 2 shows polarization diagrams (**PDs**, or hodograms) in the horizontal plane of a zero-offset VSP. **PDs** of the in-line (x-) source (the particle motion of s_{11} and s_{21} , 2a), and **PDs** of the cross-line (y-) source (particle motion of s_{12} and s_{22} , 2b) are very elliptical and shear-wave splitting is difficult to identify. But almost all **PDs** of the transformed motions, $V2(t)$ (particle motion of $\xi = s_{11} - s_{22}$ and $\eta = s_{12} + s_{21}$, 2c) and **PDs** of $V1(t)$ (particle motion of $\zeta = s_{11} + s_{22}$ and $\chi = s_{12} - s_{21}$, 2d) are essentially linear as expected, despite the ellipticity of the original particle motion.

We also examined similar **PDs** in the horizontal plane of **offset-VSPs**, which showed similar features. In summary,

although the particle motion of shear waves as recorded are frequently elliptical, the linear transforms of the sum and difference of the split shear-waves, $V1$ and $V2$, respectively, are linear. Consequently, the time series of the split shear-waves can be directly separated by recombining $V1$ and $V2$, regardless of the orthogonality of split shear-waves.

Results

ZERO-OFFSET VSP: Results of a zero-offset VSP are presented. The results include principal time series of split shear-waves, polarizations and time delays of split shear-waves, and down well geophone orientations. Figure 3 shows some of the results: time delays, polarizations and geophone orientations. The results agree with analysis based on **PDs** by Yardley (private communication).

OFFSET VSP: Results of an offset VSP are presented, where the geophone orientation is determined from the polarizations of P-waves. Figure 4 shows some of the results: time delays, polarizations and orthogonalities. The time delays from assuming orthogonal polarizations (solid line, 4a) and non-orthogonal polarizations (dotted line, 4a) are almost identical, presumably because the polarizations are nearly orthogonal. The measured polarizations (solid line, Figure 4b) are similar to those of the zero offset VSP (Figure 3b). The orthogonality (dotted line, Figure 4b) of the two split shear-waves are about the 90° line, as expected from the comparison of the delays in Figure 4a.

REFLECTION SURVEY: Results of the Lost Hills reflection data (Squires et al., 1989) will be presented. The results include separation into principal time series of the split shear-waves, measured from shot records, colour polarizations of split shear-waves, and final stacking section of principal time series. The colour polarization section can be used to identify **lateral** variation of anisotropy and the stacking section is optimized.

Discussion

This technique recognizes the essential linearity of the shear-wave motion in the transformed coordinate system. These (four) linear transforms are deterministic and can be efficiently implemented. The technique allows unknown **downwell** geophone orientations to be used, where conventional rotation technique often fails (Figure 3). The technique also allows large offset data, where the split shear-waves may not be orthogonal in the horizontal plane (Figure 4). In reflection data, shear-wave polarization and amplitudes can be more easily measured from the transformed components. The technique can be extended to media where the effective polarizations changes with depth, and in this case, we can extrapolate the sources downwards into the lower layers, so that the medium between the extrapolated sources and the geophones is homogeneous, as was demonstrated by Winterstein and Meadows (1990).

Conclusions

Recorded shear-wave motion can be linearized in a variety of conditions by four linear transforms. This simplifies the processing of four-component shear-wave **VSPs** or reflection surveys in the presence of anisotropy, and allows various attributes measured, regardless of the orthogonality of the split shear-waves. Comparing with conventional techniques, this linear transform technique is computing fast, flexible, and widely applicable.

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FIG. 1. Diagrams showing the acquisition geometry and coordinate system in the horizontal plane. (a) Source geometry, where x and y are two orthogonal sources with signature $F(t)$, and F_1 and F_2 are two decompositions of the source vector F . (b) Geophone geometry, where x and y are two orthogonal geophones possibly in a different orientation from the source signals.

FIG. 2. Polarization diagrams (PDs) of a zero-offset VSP from BP, showing the linearity of the transformed motions: (a) in-line source (x -source), $s_{11}(t)$ and $s_{21}(t)$; (b) cross-line source (y -source), $s_{12}(t)$ and $s_{22}(t)$; (c) transformed components, $\xi(t) = s_{11}(t) - s_{22}(t)$ and $\eta(t) = s_{12}(t) + s_{21}(t)$; and (d) transformed components $\zeta(t) = s_{11}(t) + s_{22}(t)$ and $\chi(t) = s_{12}(t) - s_{21}(t)$.

FIG. 3. Results of the zero-offset VSP in Figure 2. (a) measured time delays, (b) measured polarization direction of faster split shear waves; and (c) comparison of geophone orientations (x-direction) estimated by the linear transform technique (solid line) and by visual examination of polarization diagrams, PDs (dotted line), where both attributes are measured from the source x-direction.

FIG. 4. Results of an offset VSP from BP. (a) comparison of time delay measured using the orthogonal algorithm (solid line); and using the non-orthogonal algorithm (dotted line); and (b) measured orthogonality, angle between the two split shear-waves (dotted line), and polarization angles of the faster split shear-waves (solid line), where both attributes are measured from source x-direction.