Using a semi-empirical approach for petrophysical inversion: a case study from the North Sea

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Summary

We present an approach which combines with the theoretical models of Xu-White (1995) and empirical model of Han *et al.*(1986) for velocity prediction in sand-clay environments. The main goal is to provide an practical relationship between velocity and reservoir parameters such as porosity, clay content and pore aspect ratio. This may then be used for petrophysical interpretation of P- and S-wave velocities and inversion for aspect ratio. To achieve this, firstly, we use Kuster-Toksoz theory (1974) proposed by Xu-White (1995) to derive bulk and shear moduli and velocities in sand-clay mixture. Secondly, we use this result in an artificial neural network to predict the variation of aspect ratio with depth based on log data. Finally, we use the variable aspect ratios to link the model of Han *et al*(1986) to form a new semi-empirical model for petrophysical interpretation. The new approach provides a key to obtaining a more consistent relationship of velocities with reservoir parameters. Tests on the North Sea data show that this approach can provide satisfactory results for velocity predictions.

Introduction

In order to describe the distribution of reservoir parameters, various theoretical models have been proposed, which attempt to link porosity, fluid, clay content with matrix elastic rock properties (Gassmann, 1951; Kuster-Toksoz,1974; Mavko and Toksoz, 1974 and Han *et al*,1986). Although, all these theories give approximately the same prediction for velocities in porous rocks, they all have a different physical basis and application conditions, and none of them includes the effect of pore aspect ratio (the ratio of short axis to long axis) on velocities in sand-clay environments. Xu-White (1995) applied the Kuster-Toksoz (1974) and Gassmann (1965) models to predict P- and S-wave velocities in sand-clay mixture; their results suggest that the effects of aspect ratio can explain most of the scattering in the porosity-velocity relation. Although Xu and White pointed out that aspect ratio may significantly affect elastic moduli and velocities, they only use a couple of fixed aspect ratios in their model (for example, using 0.1 for sand and 0.02 for clay). This works well in consolidated formations, but it is not suitable for rocks with loose matrix and fractures, or rocks containing fluids, such as gas and live oil. Han *et al*(1986) use the method of data-calibration under formation condition, which works in the loose matrix, even with fluids. However there is a sacrifice in accuracy. This is because there is a quite complex relationship between velocities and aspect ratios (see Fig.1). To overcome these weakness, we propose a new approach combining the methods of Xu-White (1995) and Han et al (1986). In this new model, the pore aspect ratio is introduced into the model of Han *et al* using a multiple regression.

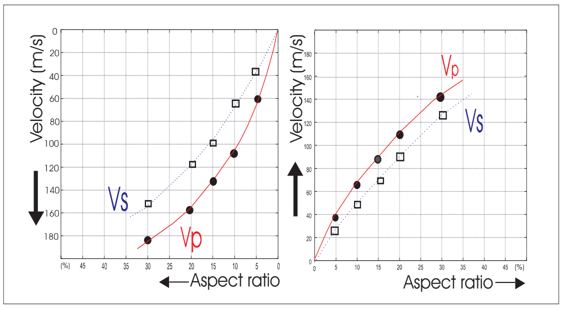

Figure 1. Relationship of velocities with aspect ratio (based on Xu-White model, data from the North Sea)

Methodology

Firstly of all, the Xu-White model is used to derive preliminary elastic moduli and predicted P- and S-wave velocities. Secondly, the measured velocity from dipole sonic log will be employed to model values for comparison with predicted values. New aspect ratios over the entire depth range are inverted using a back-propagation neural network, and the new aspect ratios are then input to the modified Xu-White model to replace the fixed aspect ratio and output new predicted velocity. The new aspect ratios are also used in the model of Han *et al* to link with the Xu-White model by means of a multiple regression.

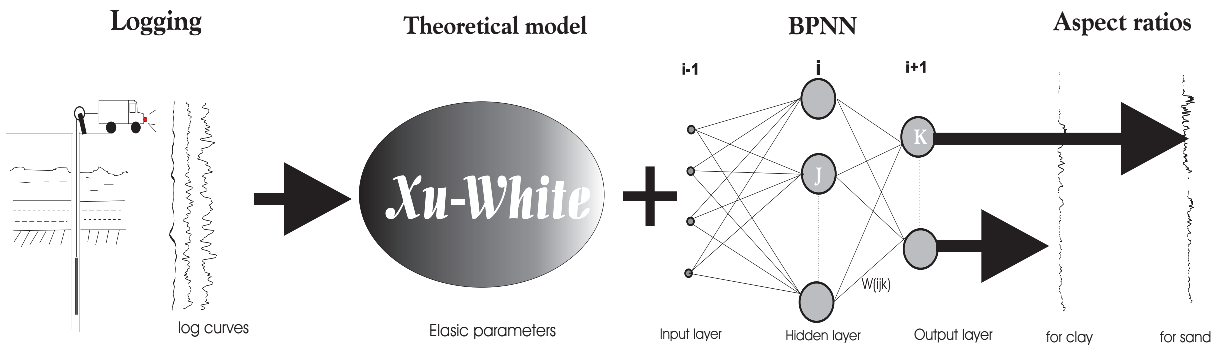

Figure 2. An idealised artificial neural network (BPNN) for inversion of aspect ratios in sand-clay mixture.

Xu-White model: The Xu-White model (1995) is based on the Kuster-Toksoz(1974) and Gassmann (1965) models. Kuster-Toksoz (1974) give a series of equations to determine elastic moduli; these include bulk and shear moduli for dry frame (K_d and μ_d), mixture (K_m and μ_m), fluid (K_f and μ_f), P- and S-wave velocity (V_p and V_s). The main relationship between the aspect ratio and elastic moduli is given by

$$K_d - K_m = \frac{1}{3}(K_f - K_m)\frac{3K_d + 4\mu_m}{3K_m + 4\mu_m}) \sum_{l=s,c} \Phi_l T_{iijj}(\alpha_l), \tag{1}$$

$$\mu_m - \mu_m = \frac{(\mu_f - \mu_m)}{5}\frac{6\mu_d(K_{m+2\mu_m}) + \mu_m(9K_m + 8\mu_m)}{5\mu_m(3K_m + 4\mu_m)} \cdot$$

$$\cdot \sum\nolimits_{l=s,c} \Phi_l \left(T_{iijj}(\alpha_l) - \frac{T_{ijij}(\alpha_l)}{3}\right), (2)$$

Where, Φ is total effective porosity of the mixture, $T_{iijj}(\alpha_l)$ and $T_{ijij}(\alpha_l)$ are the functions of aspect ratio for sand and clay.

Artificial neural network: Xu-White consider that the porosity can be divided into components attached to each clay and sand fraction separately, and that these fractions should possess different aspect ratio $\mathrm{T}_{iijj}(\alpha)$ and $T_{ijij}(\alpha)$. Because $T_{iijj}(\alpha)$ and $T_{ijij}(\alpha)$ cannot be measured directly from field data, in the previous application of published papers, the aspect ratios were always fixed, and these fixed values were often used for large depth intervals up to several hundred meters. In reality, the pore aspect ratios are not evenly distributed, and their values vary with lithology, fluid and other factors. In the new approach, this restriction is relaxed by the construction of an inversion procedure using a back-propagation neural network (BPNN). It is based on simple linear processing elements which interact to form complex non-linear behaviour, and can learn to recognise patterns in data and develop their own generalisations. To calculate the average value of error (ΔE_n) of aspect ratio in the input pattern, the weight values are adjusted based on the previously determined values until all training sets are exhausted. It is also necessary to adjust the studying rate(η) during training based on

$$\Delta w_{ijk}(n+1) = \eta\delta_{jk}o_{ij+}\alpha\Delta w_{ijk}(n), \tag{3}$$

where, Δw_{ijk} is the variation of weight connection, η is the learning rate, α is the momentum rate, o_{ij} is the output, and δ_{ik} is the error variation. A diagrammatic model with an idealised artificial neural network is shown in Figure 2.

Han et al (1986) and new extension: For estimating the influence of porosity (ϕ) and clay (V_{cl}) on seismic velocity (V), Han *et al*(1986) used least square fitting of a series of borehole or laboratory measurements to polynomial in the form of

$$V = f\,[\phi,\, V_{cl},\,]. \tag{4}$$

In particular, the data of Han *et al.* (1986) measured at the confining pressure of the formation and porosity condition reveal a relationship between velocity, porosity and clay. This relationship implies that the P- and S-wave velocities are the function of porosity and clay content. In our approach, we use the forward segmentation multiple regression to extend the empirical model of Han *et al*(1986), and combine it with Xu-White model by the aspect ratio as follows;

$$V = f\,[\phi,\, V_{cl},\, T(\alpha)]. \tag{5}$$

Therefore, velocity, porosity, clay content and aspect ratios are combined together as the regression elements. We can then obtain an optimum fitting of P-wave velocity (V_p) and S-wave velocity (V_s) as follows

$$V_p = A_0 + A_1 \cdot \phi + A_2 \cdot Vcl + \sum_{l=s,c} A_l \cdot T_l(\alpha), \tag{6}$$

$$V_s = B_0 + B_1 \cdot \phi + B_2 \cdot Vcl + \sum_{l=s,c} B_l \cdot T_l(\alpha). \tag{7}$$

Where, $\sum_{l=s,c} T_l(\alpha)$ are the aspect ratios of sand and clay respectively; parameters A_0, A_1, A_2, A_l and B_0, B_1, B_2, B_l will be calculated through multiple regression for the different areas.

Application

We test this method on field data from the North Sea. The input data include log and core data, and the log data consist of caliper log (CAL), gamma-ray log (GR), density log (DEN), self-potential log (SP), and dipole sonic log (DT). The core data include porosity(Φ) and clay content (V_{cl}) derived in the laboratory. The fitting equations are based on equation **??** and **??** as follows

$$V_p = 4.198 - 4.17\phi - 1.64Vcl +$$

$$+[2.99\mathrm{T}_s(\alpha) - 4.05T_c(\alpha)], (8)$$

$$V_s = 3.199 - 3.24\phi - 1.42Vcl +$$

$$+[2.64\mathrm{T}_s(\alpha) - 3.32T_c(\alpha)], (9)$$

where, velocities are expressed in Km/s. The parameters of the semi-empirical model are explained in Figure 3.

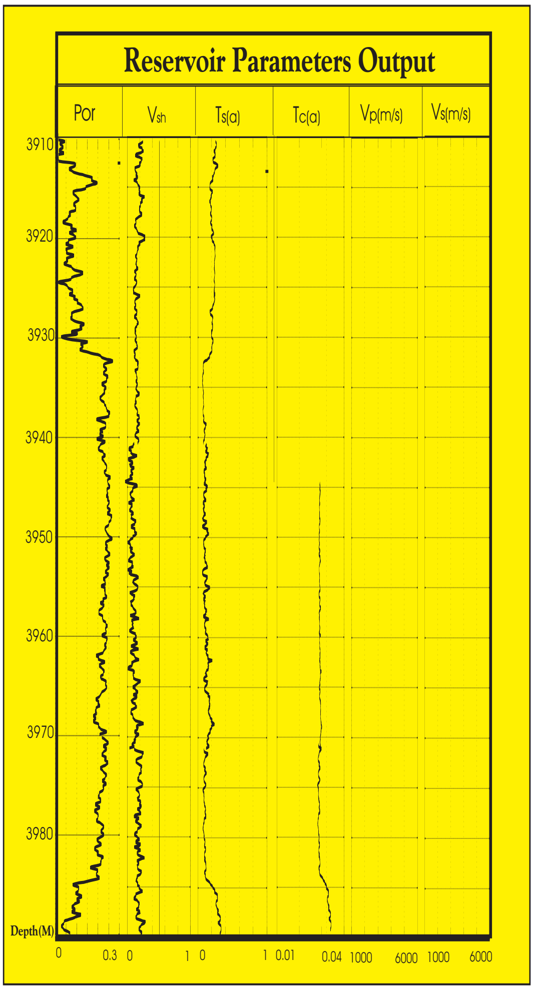

Figure 3. Reservoir parameters outputs in the new combined model (data from the North Sea)

Figure 4 illustrates error comparison of the predicted and measured velocities for Vp and Vs using different models. These models include the model of Han *et al*; original Xu-White model with fixed aspect ratio; modified Xu-White model with variable aspect ratio and our semi-empirical model. The results show that our approach is at least as accurate as others. The error analysis between prediction and measurement also confirms the results.

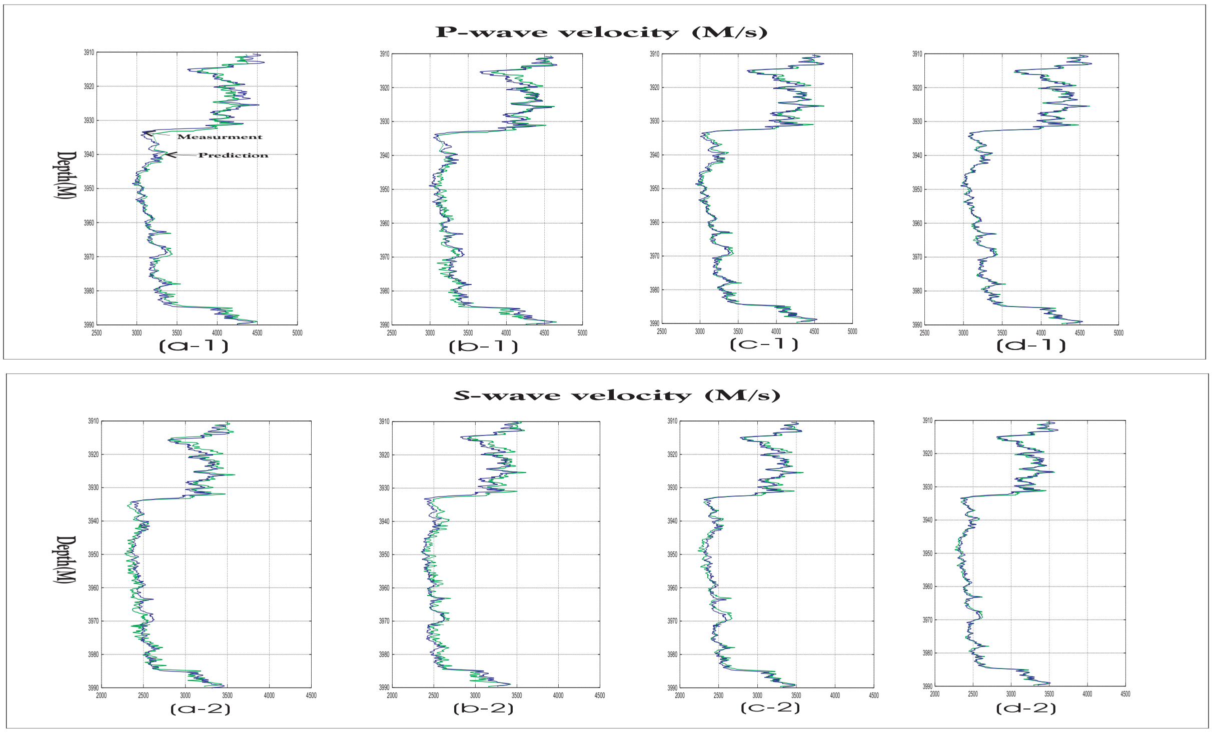

Figure 4. Comparison of prediction and measurement P- and S-wave velocities in different models. (a-1)and (a-2) are Han's *et al* model; (b-1) and (b-2) are original Xu-White model; (c-1) and (c-2) are modified Xu-White model; (d-1) and (d-2) are our semi-empirical model.

Conclusions

We have proposed a new approach for velocity prediction and petrophysical inversion, which combines the Xu-White theoretical model (1995) and Han's *et al.*(1986) empirical model. The advantage of this approach is to use pore aspect ratios to characterize the compliances of the sand and clay components. This approach is not only used to estimate the relationship of reservoir parameters with elastic properties but also can be extended to predict velocity and invert pore aspect ratio based on log and core data. Although this approach uses semi-empirical equations, the validity of this approach is confirmed by field data and error analysis.

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