Incorporating stress-sensitivity into dynamic poro-elasticity.

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1 Summary

Attempts to relate high frequency laboratory measurements to low frequency field data are typically impeded by significant frequency dispersion between the two cases. We derive a new poro-elastic model on the basis of Eshelby theory and calibrate it against resonant bar and ultrasonic measurements of Vp, Vs and Qs over an extended range of frequencies and for two values of effective stress. We advance a quantitative interpretation of the data in terms of fluid flow and viscous relaxation mechanisms.

2 Introduction

An understanding of the processes associated with the dispersion of seismic waves is essential if low frequency field data is to be interpreted in light of high frequency laboratory measurements. The theory of poro-elasticity (Biot, 1956) gives a dispersion equation for fluid saturated rock but generally underestimates the magnitude of the dispersion effects. More recent advances (Dvorkin *et al.*, 1995) have incorporated the concepts of "squirt flow" and "soft porosity" into the original Biot formulation and have produced more realistic dispersion equations.

The interpretation of seismic anisotropy in terms of stress-aligned micro-cracks (APE-modellling, Zatsepin & Crampin, 1997) relies on crack based velocity models which follow from the work of Eshelby (1957). A common assumption in these models is that the cracks are isolated from each other with respect to fluid flow at the time scale of the wave , implying a complete absence of dispersion. Attempts to put fluid flow into the crack models typically either neglect overall mass conservation (O'Connell & Budiansky, 1977) leading to results with a continuous range of characteristic frequencies and essentially an infinite number of free parameters or rely on problematic averaging procedures (Hudson *et al.*, 1996). None of these models is able to reproduce the main results of Biot theory, for example the existence of a slow P-wave. In this paper we derive a poro-elastic model based on Eshelby theory and calibrate it against laboratory data.

3 Theory

We consider pore space to consist of a random lattice with either a thin crack or a spherical pore at each vertex. Fluid is assumed to flow between neighbouring vertices due to pressure differences following the equation:

$$\partial_t m_a = \frac{\rho_f k \lambda}{\eta}(p_b - p_a), \tag{1}$$

where m_a is the mass in element a, ρ_f is the fluid density, k is permeability, λ is the grain size, η the fluid viscosity and p_a the pressure in element a.

When a stress wave propagates, different pressures will be induced in elements according to their geometries, orientations and positions along the axis of wave propagation. Applying the expectation operator to equation (1) and solving gives the result:

$$p_i = \frac{\sigma_i}{1+K_c} + \gamma p^* - \gamma' \sigma - \frac{1}{\tau}\int_0^t [\frac{\sigma_i(s)}{1+K_c} + (\gamma - 1)p^* - \gamma'\sigma(s)]e^{\frac{s-t}{\tau}} ds, \quad (2)$$

where p_i is the pressure in a crack with orientation i, σ_i is the normal stress acting on a crack with orientation i, p^* is the expected pressure in a spherical pore, σ is the trace of the stress wave, τ is the characteristic relaxation time, and K_c, γ and γ' are dimensionless constants. This means that the pressure in a crack of arbitrary orientation may be written in terms of σ and p^*. However, σ and p^* are found to be coupled by the equation:

$$\gamma \ddot{p^*} + \frac{1}{\tau}(\iota + (1-\iota)\gamma)\dot{p^*} - \frac{\lambda^2}{6\tau}(1 - \iota(1-\gamma))\dot{p^*}'' - \frac{\lambda^2}{6\tau^2}p^{*''} = \gamma'\ddot{\sigma} + \frac{1}{\tau}[\frac{\iota}{3(1+K_c)} + (1-\iota)\gamma']\dot{\sigma} + \frac{\lambda^2 \iota}{6\tau}(\frac{1}{3(1+K_c)} - \gamma')\dot{\sigma}'', \quad (3)$$

where ι is a number between 0 and 1 describing the number of cracks relative to the number of pores. Now following Eshelby's Interaction Energy approach, the effective elastic constants can be found if we know the pressure and strain inside each inclusion. These may be given in terms of σ and p^*, so the general wave equation gives a second equation coupling p^* and σ. This equation is solved together with (3) to give a dispersion equation. The model predicts the existence of 2 P-wave velocities in agreement with Biot's theory.

For shear waves we have the effective shear modulus,

$$\mu_{eff} = \mu - \frac{1}{6}\phi_c \frac{1}{1+K_c}\frac{\mu^2}{\sigma_c}[K_c + \frac{1}{1+i\omega\tau}] - \frac{1}{2}\phi_c \frac{\mu}{\frac{i\omega\eta}{\mu - i\omega\eta} + \frac{2-v}{1-v}\frac{\pi}{2}r} - 15\phi_p \mu \frac{1-v}{7-5v}[1 - \frac{i\omega\eta}{\mu}], \quad (4)$$

where ω is the frequency, μ is the mineral shear modulus, ϕ_c the fractional volume of the microcracks, ϕ_p that of the pores, v is the Poisson's ratio, r the crack aspect ratio and σ_c the critical stress (Zatsepin & Crampin 1997).

4 Modelling

The experimental work of Sothcott *et al.* (2000) has given new insights into the nature of dispersion in fluid saturated rocks. Resonant bar and ultrasonic measurements were taken on a sample of reservoir sandstone saturated with brine at a range of effective stresses. We now use some of their results to calibrate the theoretical model. Figure 1 shows the results of our modelling. The shear velocity curves were constructed by choosing appropriate values for crack density and the relaxation time. For 40 MPa effective stress a smaller crack density was used (.04 as opposed to .16 for 20 MPa), modelling the closure of microcracks. The shear wave Quality factor curves follow when we supply a stress sensitive constant Q component to model frictional attenuation. The P-wave velocity curves require the specification of the constant ι, which must of course be lower for 40 MPa effective stress than for 20 MPa. The data for P-wave Quality factor was rather noisy so we have not attempted to model it.

In all cases the behaviour below 100 kHz is due to fluid flow, while above 100 kHz the main mechanism is viscous relaxation of the fluid within individual cracks. The modelling for this latter mechanism depends on knowing the aspect ratio spectrum, which clearly we do not know, so the behaviour of the curves above 100 kHz should only be viewed as crude qualitative estimates. It should be noted that this restriction does not apply to modelling the fluid flow at sonic frequencies.

The crack densities we have used may be inverted for the crack space compressibility parameter common in APE-modelling. We find the value to be .065 MPa^{-1} which is very close to the values usually found for this type of rock by purely ultrasonic means. The model predicts that the slow P-wave has a velocity of approximately 70 ms^{-1} at 1 MHz and a Quality factor of around 1.

5 Conclusions

Our model is capable of explaining the experimental results of Sothcott *et al.* (2000). We conclude that at sonic frequencies the most important mechanism is fluid flow, while viscous relaxation effects dominate at ultrasonic frequencies. The similarity of the crack space compressibility parameter inferred from the new model to those arising from previous work enhances our confidence in the approach.

6 References

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7 Acknowledgements

The authors thank Jeremy Sothcott and Clive McCann for allowing us to view the data in advance of publication and for permission to reproduce it in this abstract.

Stress-sensitive poro-elasticity

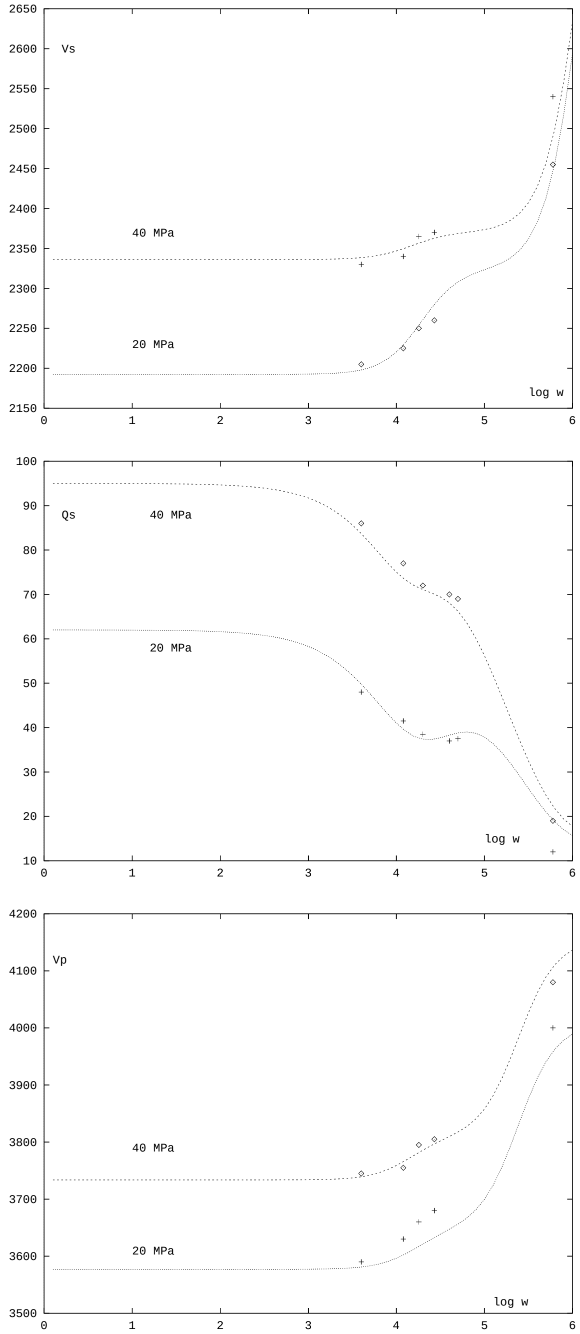

Fig1: Modelling of Vs, Qs and Vp as a function of frequency for 20 MPa and 40 MPa effective stress. Data courtesy of J. Sothcott.